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ANALYSIS OF THE EFFICIENCY IN CERTAIN MASS-EXCHANGE MODELS WITH DIFFERENT EQUILIBRIUM DEPENDENCES

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The efficiency of mass exchange in the complex and hypothetical models and in the Murphree and Hausen models for concurrent and cross motions of the vapor and liquid phases has been compared with the use of equilibrium dependences of the y = mx and y = ax + b type. Industrial data on separation of furfurol–water and ethanol–water binary mixtures have been employed in the calculations. Satisfactory convergence of the results has been obtained.

In [1], based on the experimental data on separation of a benzene–toluene mixture, we have compared the efficiency of mass exchange for two variants of the equilibrium-line equation

$$y = mx \,, \tag{1}$$

$$y = ax + b \,. \tag{2}$$

When the change in the concentration of the components is considerable, the use of formula (1) leads to substantial errors and it is more preferable to employ formula (2) ensuring satisfactory convergence of the values of the efficiencies, which have been calculated for highly volatile and involatile components. In the present work, a study is made of the possibility of employing (1) and (2) for small concentration changes of the components on the plate.

From [2, 3], we know the expressions of equilibrium lines for ethanol–water and furfurol–water binary mixtures respectively

$$m_{\rm e} = 0.277 \cdot \frac{2.5 - x}{0.053 + x} + 0.6x \,, \tag{3}$$

$$m_{\rm f} = 2 \cdot \frac{0.12 - x}{0.03 + x} \,, \tag{4}$$

in which the highly volatile component is represented by mole fractions.

We reduce formulas (3) and (4) to the form (1):

$$y_{\rm e} = 0.277 \cdot \frac{2.5 - x}{0.053 + x}\, x + 0.6x^2 \,, \tag{5}$$

$$y_{\rm f} = 2 \cdot \frac{0.12 - x}{0.03 + x}\, x \,. \tag{6}$$

For transformation of Eqs. (5) and (6) to the structure (2) we have found derivatives and have determined the corresponding slopes of the equilibrium curves a:

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TABLE 1. Results of Calculation of the Beer (n = 20) and Alcohol (n = 15) Columns of the Bobruisk Hydrolysis Plant

n	x_{bottom}, wt.%	x_{top}, wt.%	L/V	E	E_1	E_2	E_3	E_4
Concurrent flow $y = mx$								
20	0.01736	0.8457	6.5	0.3735	0.1787	0.3005	0.3929	–
15	0.01527	20.0	2.5586	0.5274	0.1939	0.4960	0.5502	–
$y = ax + b$								
20	0.01736	0.8457	6.5	0.3686	0.1764	0.2938	0.3866	–
15	0.01527	20.0	2.5586	0.4929	0.1807	0.4415	0.5137	–
Cross flow $y = mx$								
20	0.01736	0.8457	6.5	0.2410	–0.0274	0.1503	0.2625	–0.2149
15	0.01527	20.0	2.5586	0.2950	–0.1405	0.2480	0.3291	–0.4920
$y = ax + b$								
20	0.01736	0.8457	6.5	0.2397	–0.0001	0.1486	0.2615	–0.2113
15	0.01527	20.0	2.5586	0.2785	–	0.2195	0.3193	–0.3913

$$a_{\text{e}} = \frac{dy_{\text{e}}}{dx} = 0.277 \cdot \frac{0.1325 - 0.106x - x^2}{(0.053 + x)^2} + 1.2x\,, \tag{7}$$

$$a_{\text{f}} = \frac{dy_{\text{f}}}{dx} = \frac{0.0072 - 0.12x - 2x^2}{(0.03 + x)^2}\,. \tag{8}$$

By simultaneous solution of (2), (5), and (7) and of (2), (6), and (8) we have obtained the segments b cut off by the corresponding tangent lines on the ordinate axis:

$$b_{\text{e}} = \frac{0.7072x^2}{(0.053 + x)^2} - 0.6x^2\,, \tag{9}$$

$$b_{\text{f}} = \frac{0.3x^2}{(0.03 + x)^2}\,. \tag{10}$$

The calculated dependences obtained have been checked with the use of industrial data of the main furfurol column of the Bendery Biochemical Plant and the beer and alcohol columns of the Bobruisk Hydrolysis Plant. Expressions for the cross motion of the vapor and liquid phases and for the concurrent flow where the calculated dependences correspond to the complete mixing of the liquid on the plate were employed.

From [4] with account for [5] the difference of the concentrations of the highly volatile component on the plate in concurrent flow in the liquid and vapor phases can be represented in the following form:

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \dfrac{y_{n-1} - b}{m}\right) E_{\text{con}}}{(1 - h)\,\dfrac{L}{mV} + 1 - h_1 + h\,\dfrac{L}{mV}\,E_{\text{con}} - (1 - h_1)\,E_{\text{con}}}\,, \tag{11}$$

$$y_n - y_{n-1} = \frac{\left(x_{n-1} - \dfrac{y_{n-1} - b}{m}\right) \dfrac{L}{V}\,E_{\text{con}}}{(1 - h)\,\dfrac{L}{mV} + 1 - h_1 + h\,\dfrac{L}{mV}\,E_{\text{con}} - (1 - h_1)\,E_{\text{con}}}\,. \tag{12}$$

TABLE 2. Results of Calculation of the Main Furfurol Column of the Bendery Biochemical Plant

x_{top}, wt.%	x_{bottom}, wt.%	L/V	E	E_1	E_2	E_3	E_4
Concurrent flow $y = mx$							
0.30	0.022	4.974	0.2424	0.1220	0.1812	0.2648	–
0.35	0.023	4.000	0.2053	0.0889	0.1620	0.2254	–
0.38	0.024	4.470	0.2313	0.1090	0.1780	0.2531	–
0.37	0.027	4.470	0.2175	0.1016	0.1667	0.2384	–
0.20	0.010	3.630	0.2098	0.0856	0.1703	0.2302	–
0.26	0.014	3.765	0.2105	0.0880	0.1692	0.2309	–
0.56	0.037	3.882	0.2009	0.0856	0.1596	0.2209	–
0.51	0.030	3.529	0.1950	0.0778	0.1587	0.2144	–
0.53	0.037	4.470	0.2229	0.1049	0.1711	0.2444	–
$y = ax + b$							
0.30	0.022	4.974	0.2410	0.1219	0.1799	0.2637	–
0.35	0.023	4.000	0.2041	0.0889	0.1607	0.2242	–
0.38	0.024	4.470	0.2295	0.1090	0.1764	0.2518	–
0.37	0.027	4.470	0.2161	0.1015	0.1652	0.2371	–
0.20	0.010	3.630	0.2092	0.0857	0.1696	0.2255	–
0.26	0.014	3.765	0.2096	0.0880	0.1682	0.2301	–
0.56	0.037	3.882	0.1989	0.0856	0.1574	0.2109	–
0.51	0.030	3.529	0.1932	0.0777	0.1568	0.2128	–
0.53	0.037	4.470	0.2209	0.1048	0.1688	0.2425	–
Cross flow $y = mx$							
0.30	0.022	4.974	0.1586	0.0249	0.0906	0.1834	–0.1107
0.35	0.023	4.000	0.1285	0.0009	0.0810	0.1505	–0.0967
0.38	0.024	4.470	0.1481	0.0125	0.0890	0.1723	–0.1083
0.37	0.027	4.470	0.1392	0.0117	0.0833	0.1623	–0.1000
0.20	0.010	3.630	0.1287	–0.0082	0.0852	0.1511	–0.1026
0.26	0.014	3.765	0.1301	–0.0049	0.0846	0.1525	–0.1018
0.56	0.037	3.882	0.1251	–0.0029	0.0798	0.1469	–-0.0949
0.51	0.030	3.529	0.1191	–0.0092	0.0794	0.1404	–0.0943
0.53	0.037	4.470	0.1428	0.0125	0.0855	0.1664	–0.1031
$y = ax + b$							
0.30	0.022	4.974	0.1579	0.0256	0.0899	0.1830	–0.1096
0.35	0.023	4.000	0.1274	0.0018	0.0802	0.1500	–0.0956
0.38	0.024	4.470	0.1474	0.0013	0.0882	0.1717	–0.1071
0.37	0.027	4.470	0.1385	0.0127	0.0825	0.1617	–0.0989
0.20	0.010	3.630	0.1284	–0.0078	0.0848	0.1508	–0.1022
0.26	0.014	3.765	0.1297	–0.0042	0.0841	0.1522	–0.1011
0.56	0.037	3.882	0.1241	–0.0008	0.0787	0.1461	–0.0934
0.51	0.030	3.529	0.1182	–0.0080	0.0784	0.1396	–0.0930
0.53	0.037	4.470	0.1419	0.0139	0.0844	0.1656	–0.1016

Such dependences have also been derived for the cross motion of flows with account for [5, 6]

$$x_n - x_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1} - b}{m}\right) E_\text{k}}{(1-h)\frac{L}{mV} + \frac{1}{2} - h_1 + h\frac{L}{mV} E_\text{k} - (1-h_1) E_\text{k}}, \tag{13}$$

$$y_n - y_{n-1} = \frac{\left(x_{n-1} - \frac{y_{n-1} - b}{m}\right) \frac{L}{V} E_\text{k}}{(1-h)\frac{L}{mV} + \frac{1}{2} - h_1 + h\frac{L}{mV} E_\text{k} - (1-h_1) E_\text{k}}. \tag{14}$$

In the calculations, we employed the following models differing, in formulas (11)–(14), in the distances h and h_1: the complex model in which $h = h_1 = 1/(m+1)$, the Murphree model in analyzing the efficiency in the vapor phase ($h = 0$, $h_1 = 1$, the 1st variant) and in analyzing the efficiency in the liquid ($h = 1$, $h_1 = 0$, the 2nd variant), the Hausen model ($h = h_1 = 0$, the 3rd variant), and the hypothetical model ($h = h_1 = 1$, the 4th variant).

As the initial data we took the ratio of the molar flow rates L/V, the concentrations of the highly volatile component in the cube product x_bottom and on the feed plate x_top, the zero concentration of the highly volatile component in the vapor coming from the cube since the columns are heated by a live steam, and an arbitrary value of the efficiency.

At first, we computed the coefficients of phase equilibrium m or the quantities a and b for $x = x_{n-1}$. Next, for an arbitrary efficiency, we calculated the compositions of the liquid arriving at the first plate at the bottom and of the vapor escaping from it from (11) and (12) or (13) and (14) with account for the distances h and h_1 for each variant. From the new values of x we refined the values of m for the second plate at the bottom and found the corresponding concentrations for it. Analogous calculations were carried out for all the plates, including the feed plate. In the case of a difference between the calculated concentration of the highly volatile component in the liquid arriving at the feed plate and its experimental value, we corrected the new value of the efficiency and repeated the calculations until the concentration of the volatile component in the liquid arriving at the upper plate converged.

The calculation results are given in Tables 1 and 2.

Analysis of the calculation results yields that the values of the efficiency in the case of employment of the structures of formulas (1) and (2) differ, on the average, by several percent for all the employed models of mass exchange and forms of organization of the flows.

For high values of the coefficient m the efficiency in the complex model is between the efficiencies in the models of Murphree (the second variant of mass exchange) and Hausen. With increase in m the complex model approaches the latter. For lower values the efficiency in the complex model is intermediate between the efficiencies in the Murphree model of the two variants. The above features are characteristic of both the concurrent and cross motions of the interacting phases.

In concurrent flow, the efficiencies in the vapor phase are 1.5–2 times higher than the corresponding values in cross flow in all the models considered except for the Murphree model; this relation is equal to two for all the columns and their parts for the efficiency in the liquid. The negative values in cross flow in the Murphree model dropping out from the regularity indicated stem from the small L/V ratios (alcohol column), which reduces the usefulness of this variant.

Thus, the employment of the equilibrium-line equations in the form of formulas (1) and (2) in the case of small concentration changes of the components on the mass-exchange plate virtually leads to the same results. At the same time, taking into account the results of [1], one should consider expression (2) as being more universal, since its use ensures the minimum computational errors.

NOTATION

a, slope of the equilibrium curve; b, segment cut off by the tangential line to the equilibrium line on the ordinate axis; E, efficiency of mass exchange; h and h_1, dimensionless distances from the site of injection of the vapor and the liquid respectively to the surface of equality of concentrations in ideal and real plates; L, molar flow of the liquid; m, coefficient of phase equilibrium; V, molar flow of the vapor; x and y, concentrations of the highly volatile component in the liquid and the vapor phase. Subscripts: top, refers to the feed plate; k, cross current; bottom, refers to the column's cube; n, No. of the plate in question; $n - 1$, No. of the preceding plate in the direction of motion of the vapor; con, concurrent flow; f, presence of furfurol in the mixture; e, presence of ethanol in the mixture.

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